CHARACTERIZATION OF RAIN-WIND INDUCED STAY-CABLE VIBRATIONS FROM FULL-SCALE MEASUREMENTS

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Abstract

A characterization of rain-wind induced vibrations is presented from analysis of vibration records obtained from a long-term field measurement program at the Fred Hartman Bridge in Houston, Texas. Displacement time histories have been generated by filtering and numerically integrating recorded cable accelerations. An analysis of modal participation is presented, along with correlations with wind speed and direction. Values of negative aerodynamic damping have been estimated from records of incipient vibration, suggesting the levels of supplemental damping necessary for mitigation of rain-wind induced vibrations.

INTRODUCTION

Stay cables have very low levels of inherent damping, rendering them susceptible to multiple types of excitation [1]. Rain-wind induced vibrations [2] are among the most problematic types of vibration, as they are associated with very large amplitudes of oscillation and can occur relatively frequently. These large-amplitude vibrations are of concern because they induce undue stresses and fatigue in the cables themselves and in the connections at the bridge deck and tower, and mitigation of these vibrations has become a significant consideration in cable-stayed bridge design and for retrofit of existing structures.

In order to improve the understanding of the relevant excitation mechanisms and to design effective and economical systems for vibration mitigation, it is important to have a comprehensive characterization of the vibrations occurring in the field. Much of the previous research on the problem of rain-wind induced vibrations has been conducted using wind tunnels, and there have been relatively few opportunities to measure the vibrations at full-scale. This paper presents a characterization of rain-wind induced vibrations from analysis of vibration events recorded during a long-term field measurement program at the Fred Hartman Bridge, a twin-deck cable-stayed bridge over the Houston Ship Channel in Texas. The Hartman instrumentation system [3] was installed on the bridge and began collecting data in October 1997. The onsite computer continuously monitors output from the transducers, and whenever predetermined thresholds in wind speed and cable acceleration are exceeded, the system triggers to stream files to disk, sampling all 64 channels at 40 Hz for 5 minutes. Approximately 2700 five-minute records were generated during the first year of monitoring, before the installation of a temporary restraining system.

VIBRATION CHARACTERIZATION

Accelerometers were used to record the cable vibrations, and as part of the previous analyses of the field-measured data, statistics for cable acceleration, wind speed and direction, and rainfall were routinely computed for each record. These statistics were stored in a relational database to facilitate
analysis of correlations among the various statistical quantities. Preliminary observations of the vibration characteristics and their correlation with wind speed, wind direction, and rainfall have been presented using the statistics in this database [3]. While these analyses have been helpful in clarifying the nature of the measured vibrations, the displacement amplitudes are often of more direct interest than the acceleration amplitudes, which exaggerate the participation of higher modes. Particularly when investigating modal participation, it is useful to consider displacement amplitudes. Consequently, a primary component of the present characterization effort is a consideration of the displacement response statistics.

The previously generated database of acceleration response statistics was used to identify records of large-amplitude rain-wind induced vibration, and segments of these records with relatively stationary amplitude and spectral content were selected for detailed analysis. Presently, 296 segments of large-amplitude vibration have been analyzed, including records from 15 different stays; analysis is ongoing and the observations presented herein are preliminary. The length of the segments selected for analysis varied according to the features of each record, with a mean length of 69 seconds and a maximum length of 140 seconds. In some records of large-amplitude vibration, the ±4g range of the accelerometers was exceeded, resulting in “saturation” of the signal; such segments were excluded from this analysis. In a few cases where most of the record was saturated, the length of the record that was usable for analysis was less than 20 seconds. Displacement time histories were generated by high-pass filtering and numerically integrating the selected acceleration record segments, and statistics were then computed from the resulting displacement record to characterize the vibration response. Fig. 1(a) shows a plot of the acceleration locus (in-plane acceleration versus lateral acceleration) for a segment of recorded vibration, and Fig. 1(b) shows the corresponding Lissajous diagram of displacement obtained from integration of the same record. The features of the displacement locus and the acceleration locus differ due to the amplification of higher modes in the acceleration signal.

The power spectral density (PSD) of each integrated displacement record was computed, and the RMS amplitude in each mode was computed by numerical integration of the PSD in the vicinity of each modal peak. The values of RMS amplitude at the accelerometer location were stored for the 5 modes with the highest amplitudes. To account for the influence of the accelerometer location, these amplitudes were scaled to peak modal amplitudes; assuming nearly sinusoidal mode shapes, the peak modal RMS amplitude in mode $i$ with RMS amplitude $a_{i,RMS}$ at the accelerometer location is given by

$$A_{i,RMS} = \frac{a_{i,RMS}}{\sin(i\pi \frac{x_m}{L})}$$

Fig. 1. Vibration Loci from a Segment of Recorded Vibration: (a) Acceleration Locus; (b) Displacement Locus
where $x_n$ is the distance of the accelerometer from the cable anchorage. The dominant mode of vibration was then identified as the mode with the highest scaled RMS amplitude. In the subsequent discussion, the mode with the highest scaled amplitude will be denoted $n$, the scaled RMS amplitude in that mode will be denoted $A_{n,RMS}$, and the corresponding frequency will be denoted $f_n$.

### Displacement Amplitudes

Fig. 2(a) shows a plot of measured values of RMS amplitude in the dominant mode at the accelerometer location, $a_{n,RMS}$, versus frequency of the dominant mode, $f_n$, with the dominant mode number indicated by the symbol. It is important to acknowledge that because the limit of ±4g in the accelerometer range was exceeded in some records of large-amplitude oscillation, only limited conclusions can be drawn about the maximum amplitudes of oscillation. Plotted with the data in Fig. 2(a) is a curve corresponding to the maximum measurable displacement for oscillations in a single mode given the limited transducer range (labeled “Measurement Limit”). Because participation of other modes tends to increase the peak accelerations, this curve provides an upper bound on the RMS displacement amplitudes that can be measured. A clear correlation is evident between the maximum measured displacements and the theoretical limit of measurement, and unfortunately, for this reason it is not possible to make reliable conclusions about the maximum amplitudes of vibration on the basis of this measured data. However, typical vibration amplitudes can be reported, and the important observation is that these amplitudes are quite large. Fig. 1(b) shows a plot of peak modal RMS amplitude in the dominant mode, $A_{n,RMS}$, scaled according to (1), versus frequency of the dominant mode, $f_n$, with the dominant mode number indicated by the symbol. The largest amplitude record analyzed thus far had a peak modal RMS amplitude of 51 cm, which corresponds to peak-to-peak oscillations of about 1.4 m.

![Fig. 2](image_url)

**Fig. 2.** RMS Displacement in Dominant Mode versus Frequency of Dominant Mode:
(a) At Accelerometer; (b) Scaled to Peak Modal Amplitude

### Modal Participation

Fig. 3 shows a histogram of the dominant mode for all the records from various stays. The number of the dominant mode ranges from 1 to 4, with most of the responses occurring in modes 2 and 3. It is important to note that because the natural frequencies are different for different stays, the frequency of oscillation may be significantly different even when the mode number is equal. Fig. 4 shows a histogram of frequency of the dominant mode, and from this figure it is evident that the responses are distributed over a fairly wide range of frequencies. Most of the responses have frequencies between 1 and 3 Hz; this is in agreement with previous observations of rain-wind induced vibrations. This information about dominant modes and frequencies of vibration is helpful in the design of vibration mitigation systems such as dampers and cross-ties.
It is also important to establish whether the vibration responses are dominated by a single mode or if multiple modes are participating in the response. To allow for a quantitative assessment, it is helpful to use a parameter called the Modal Contribution Index [4]:

\[
MCI = \frac{A_{n,RMS}}{\sqrt{\sum A_{i,RMS}^2}}
\]

In the present paper, the summation is taken over the five modes with the highest RMS amplitude. The MCI approaches unity as the response approaches a pure single mode response, and the MCI decreases as the participation of other modes becomes more significant, approaching 0.447 when the five highest modes have equal amplitude. Fig. 5 shows a histogram of MCI, and it is evident in this figure that the majority of the responses are nearly single mode responses.

The plots presented thus far have included data from all stays together; it is also helpful to consider the characteristics of modal participation for individual stays. Fig. 6 shows plots of vibration amplitude in the dominant mode versus wind speed for two different stays, with the number of the dominant mode indicated by the symbol. For each stay, most of the responses occur in a preferred mode over a fairly wide range of wind speeds, as observed previously from analysis of acceleration spectra [5]. The stays corresponding to Fig. 6 are located on opposite sides of the bridge deck and were designed with identical properties such as length, inclination angle, tension, mass per length, and diameter, and for both of these stays, mode 2 is the preferred mode. Other stays were also observed to vibrate preferentially in a specific mode over a wide range of wind speeds, and for stays with similar
characteristics, the preferred mode is often the same. It is not yet evident what determines which mode will be preferred for a given stay, but this will be a subject of further investigation.

Wind Speed and Direction

Mean values of wind speed and wind direction were also stored for each record segment. Wind speeds at deck-level are presented, and wind direction is measured in degrees clockwise from the bridge axis, with zero degrees corresponding to wind approximately from the north, directly along the bridge axis. It is evident from the measured data that wind direction has a significant influence on vibration amplitude. Fig. 7(a) shows a plot of normalized vibration amplitude versus wind direction for all stays. Because larger amplitudes are expected in the lower modes, the amplitudes have been normalized by dividing by half of the wavelength of the dominant mode, $L/n$, where $L$ is the length of the cable. It is evident that most of the responses fall into two groups: wind direction around 60 degrees and wind direction around 110 degrees. Because each stay has a different inclination angle and a slightly different orientation in plan view, it is useful to introduce a parameter that measures the direction of the wind relative to the axis of the stay. A parameter that has often been referred to in wind tunnel investigations is the “yaw angle”, $\beta^*$ [6]:

$$\beta^* = \sin^{-1}(\sin \theta \cos \alpha)$$  \hspace{1cm} (3)

where $\theta$ is the wind direction relative to the projection of the stay axis in the horizontal plane and $\alpha$ is the inclination angle of the stay. This parameter is equal to zero when the wind direction is perpendicular to the cable axis and is equal to 90° when the wind direction is parallel to the cable axis. Positive yaw angles correspond to the cable declining in the direction of the wind flow, and negative yaw angles correspond to the cable inclining in the direction of wind flow. Fig. 7(b) shows a plot of normalized vibration amplitude versus yaw angle for all stays. It is evident in this figure that most of the high amplitude responses occur over a fairly narrow range of positive yaw angles, between 0° and

Fig. 6. Displacement Amplitude vs. Wind Speed with Dominant Mode Indicated

Fig. 7. Normalized Vibration Amplitude versus (a) Wind Direction; (b) Yaw Angle
30°. This seems to confirm that the “yaw angle” is indeed a useful parameter to represent wind direction for stays wind different inclination angles.

Fig. 8(a) shows a plot of normalized vibration amplitude versus wind speed, and it is evident that the large-amplitude responses analyzed so far occur over a limited range of wind speeds, from 5.8-11.2 m/s. Not all of the measured data has yet been analyzed using the present methods, and it is known from the previous analysis that large-amplitude acceleration amplitudes have been measured over a slightly wider range of wind speeds. A reduced velocity can be defined as

\[ V_r = \frac{U}{f_n D} \]  

(5)

in which \( U \) is the mean wind speed, \( f_n \) is the frequency of oscillation (in this case, the frequency of the dominant mode), and \( D \) is the diameter of the cable. Fig. 8(b) shows a plot of vibration amplitude versus reduced velocity. The computed values of reduced velocity range from about 15 to about 95, with most of the large-amplitude responses occurring for reduced velocity values between 15 and 35. Different symbols are used to identify the dominant mode, and it is evident that using the reduced velocity parameter causes dispersion by the mode number (or the frequency of vibration). The highest values of reduced velocity are associated with mode 1 responses, and the lowest values are associated with mode 4 responses. This dispersion effect may suggest that the absolute wind speed is a more suitable parameter than the reduced velocity; it is evident in Fig. 8(a) that the responses in all modes occur over approximately the same range of absolute wind speed. A limited range of wind speeds for which large-amplitude vibrations occur may then suggest that water rivulet formation plays an important role in exciting the measured vibrations. Previous wind tunnel investigations [2] have indicated that a water rivulet forming on the upper surface of the cable plays an important role in rendering the cable aerodynamically unstable, and it has been reported that the water rivulet can only form on the upper surface of the cable over a limited range of wind speeds. The importance of water rivulet formation could then account for the observation that absolute wind velocity may be a more suitable parameter than reduced velocity.

![Fig. 8. Normalized Vibration Amplitude versus (a) Mean Wind Speed; (b) Reduced Velocity](image)

**ESTIMATION OF NEGATIVE AERODYNAMIC DAMPING**

An important characteristic of rain-wind induced vibrations is the rate of increase of vibration amplitude under the action of the excitation mechanism. If the onset of self-excited vibrations is modeled by negative damping, the oscillation amplitude increases exponentially. The net damping
ratio in mode $i$ is the sum of the mechanical damping ratio and the aerodynamic damping ratio in that mode:

$$
\zeta_{i,\text{net}} = \zeta_{i,\text{mech}} + \zeta_{i,\text{aer}} \tag{6}
$$

The mechanical damping ratio is always positive, but the aerodynamic contribution may be negative, potentially resulting in exponentially increasing oscillation amplitudes. Numerous stay vibration records have been captured in which the oscillation amplitude increases dramatically in a short period of time, and the increase in amplitude was indeed observed to be very nearly exponential. The estimated values of net negative aerodynamic damping ratio estimated from 41 records are plotted against frequency of the dominant mode in Fig. 9(a), and the values ranged from about 0.07% to almost 0.5%. These estimated values perhaps suggest the amount of additional damping that must be provided by supplemental damping devices in order to suppress the rain-wind induced vibrations. The computed values of net damping ratio can be normalized by computing the associated Scruton number:

$$
Sc = \frac{m|\zeta_{s,\text{net}}|}{\rho D^2} \tag{7}
$$

where $m$ is the mass per length of the cable, $D$ is the diameter of the cable, and $\rho$ is the density of air. Fig. 9(b) shows the Scruton Number versus frequency of the dominant mode, with the number of the dominant mode indicated by the symbol. The largest estimated value of Scruton Number was 9.1, and it is noted that all of the estimated values in Fig. 9(b) are less than 10. Based on wind tunnel data, Irwin [7] suggested that rain-wind vibrations could be avoided if the Scruton number is greater than 10, and this analysis tends to support that suggestion, indicating that adding supplemental damping to satisfy the $Sc > 10$ criterion may be sufficient for suppression of rain-wind induced vibrations. Analysis is ongoing in order to address in more detail this question of how much damping is necessary for suppression of rain-wind induced vibrations.

![Fig. 9. Estimated Values of (a) Net Negative Damping and (b) Scruton Number versus Frequency of Dominant Mode](image)

**CONCLUSIONS**

A characterization of rain-wind induced vibrations was presented using analysis of vibrations recorded during a long-term field measurement program at the Fred Hartman Bridge in Houston, Texas. Displacement time histories were generated by filtering and numerically integrating the recorded accelerations, and RMS displacement amplitudes as large as 50 cm were estimated. The dominant modes of vibration were identified from the displacement spectra, and among the 15 stays for which
records were analyzed, the number of the dominant mode ranged from 1 to 4, with most of the responses in modes 2 and 3. Most of the responses had oscillation frequencies between 1 and 3 Hz, with the largest density of responses at around 2 Hz. Most of the responses occurred almost purely in a single mode. For a given stay, the responses occurred preferentially in a specific mode over a fairly wide range of wind speeds, and the number of the preferred mode was the same for similar stays. Wind direction was observed to be an important parameter in the observed vibration responses, and the “yaw angle”, a measure of wind direction relative to the axis of the stay, was confirmed to be a useful parameter to represent wind direction for stays with wind different inclination angles, as most of the large-amplitude responses occurred over a fairly narrow range of positive yaw angles, between 0° and 30°. Values of net negative aerodynamic damping were estimated from records of incipient vibration, and the estimated value with the largest magnitude was about -0.5%, corresponding to a Scruton number of 9.1. These values may suggest the levels of supplemental damping necessary for suppression of rain-wind induced vibrations.

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